

Maxprap. Chapter wise Series Class 12th - Integration

Class : 12th

Subject: Mathematics

Max. Marks : 60

Duration : 2 Hrs

Sections: A (MCQs), B (Short Answer-I), C (Short Answer-II), D (Long Answer), E (Case-Based).

General Instructions:

- (i). All questions are compulsory.
- (ii). This question paper contains 23 questions divided into five Sections A, B, C, D and E. (iii). Section A comprises of 10 MCQs of 1 mark each. Section B comprises of 5 questions of 2 marks each. Section C comprises of 4 questions of 3 marks each. Section D comprises of 4 question of 5 marks each and Section E comprises of 2 Case Study Based Questions of 4 marks each.
- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted.

Section A

Questions 1 to 10 carry 1 mark each

1. $\int \frac{(x^2-1)}{(x^2+4)} dx = ?$

a) $x - \frac{5}{2} \tan^{-1} \frac{x}{2} + C$

b) $x - \frac{5}{2} \tan^{-1} \frac{5x}{2} + C$

c) $x + \frac{5}{2} \tan^{-1} \frac{3x}{2} + C$

d) $x - 5 \tan^{-1} \frac{x}{2} + C$

2. $\int \sqrt{x^2 + a^2} dx$ is equal to

a) $\frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \log(x + \sqrt{(x^2 + a^2)})$

b) $\frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right)$

c) $\tan^{-1}(x^2 + x + 2) + C$

d) $\frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log(x + \sqrt{(x^2 + a^2)})$

3. $\int \sqrt{\frac{x}{1-x}} dx$ is equal to

a) $\sin^{-1} \sqrt{x} + c$

b) $\sin^{-1} |\sqrt{x} - \sqrt{x(1-x)}| + c$

c) $\lim^{-1} \left(\sqrt{x(1-x)} \right) + C$

d) $\sin^{-1} \sqrt{x} - \sqrt{x(1-x)} + c$

4. $\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{\tan x}}{(\sqrt[3]{\tan x} + \sqrt[3]{\cot x})} dx = ?$

a) π

b) 0

c) $\frac{\pi}{4}$

d) $\frac{\pi}{2}$

5. $\int x \sqrt{x-1} dx = ?$

a) $\frac{2}{7} (x-1)^{3/2} + C$

b) $\frac{2}{3} (x-1)^{3/2} + C$

c) $\frac{2}{5} (x-1)^{5/2} + \frac{2}{3} (x-1)^{3/2} + C$

d) $\frac{2}{5} (x-1)^{5/2} + C$

6. $\int x^2 \sin x^3 dx = ?$

- a) $-\cos x^3 + C$ b) $-\frac{1}{2}\cos x^3 + C$ c) $\cos x^3 + C$ d) $-\frac{1}{3}\cos x^3 + C$

7. $\int x \sin x \cos x dx = ?$

- a) $-\frac{1}{4}x \sin 2x + \frac{1}{8}\cos 2x + C$ b) $\frac{1}{4}x \cos 2x - \frac{1}{8}\sin 2x + C$
 c) $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C$ d) $-\frac{1}{4}x \cos 2x + \frac{1}{8}\sin 2x + C$

8. $\int \frac{x^3}{x+1}$ is equal to

- a) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1-x| + C$ b) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1-x| + C$
 c) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1+x| + C$ d) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1+x| + C$

Assertion(A)/Reason(R) Based (9-10): In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- a) Both A and R are true and R is the correct explanation of A.
 b) Both A and R are true but R is not the correct explanation of A.
 c) A is true but R is false.
 d) A is false but R is true.

9. Assertion (A): $\int \frac{dx}{16+9x^2} = \frac{1}{12} \tan^{-1} \left(\frac{3x}{4} \right)$

Reason (R): $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$

10. Assertion (A): $\int x e^{x^2} dx = \frac{1}{2} e^{x^2} + c$

Reason (R): To solve above integral put $x^2 = t$.

SECTION – B

Questions 11 to 15 carry 2 marks each

11. Integrate the rational function $\frac{1}{x^2-9}$

12. Evaluate the integral: $\int x\sqrt{x^4+1} dx$

13. Evaluate $\int \frac{x^2}{1-x^4} dx$

14. Find: $\int \frac{\tan^3 x}{\cos^3 x} dx$

15. Evaluate $\int_0^{\pi/2} \cos^5 x dx$

SECTION – C

Questions 16 to 19 carry 3 marks each

16. Evaluate: $\int \frac{1}{(x^2+1)(x^2+2)} dx$.

17. Find: $\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$

18. Evaluate $\int_{-1}^2 |x^3 - x| dx$.

19. Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

SECTION – D

Questions 20 to 23 carry 5 marks each

20. Integrate the function $\int_1^4 [|x - 1| + |x - 2| + |x - 3|] dx$

21. Prove that: $\int_0^{\pi/2} \log (\sin 2x) dx = -\frac{\pi}{2} (\log 2)$.

22. Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x \cdot \operatorname{cosec} x} dx$

23. Evaluate: $\int x \cdot \sin^{-1} x dx$

SECTION – E (Case Study Based Questions)

Questions 22 & 23 carry 4 marks each

24. Read the following text carefully and answer the questions that follow:

Integration Symbol



Indefinite



Definite

Rajni and Priyanka practice the problems based on integrals. They will try to evaluate the integrals based upon $\int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c$.

Rajni first explains the steps to solve this type of integrals.

Step 1: Obtain the integral, let it be $I = \int \frac{f'(x)}{f(x)} dx$

Step 2: Put $f(x) = t$ and replace $f'(x)dx$ by dt to obtain $I = \int \frac{1}{t} dt$

Step 3: Evaluate integral obtained in step II to obtain $I = \log |t| + c$

Step 4: Replace t by $f(x)$ step III to get $I = \log |f(x)| + c$

- a) Evaluate: $\int \frac{2x+5}{x^2+5x-7} dx$. (1)
- b) Evaluate: $\int \frac{1}{x(3+\log x)} dx$. (1)
- c) Evaluate: $\int \frac{1}{1+e^{-x}} dx$. (2)

OR

Evaluate: $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$. (2)

25. Read the following text carefully and answer the questions that follow:

$$\int e^x [f(x) + f'(x)] dx = \int e^x f(x) dx + \int e^x f'(x) dx$$

Using integration by parts

$$= f(x) \int e^x dx - \int f'(x) e^x dx + \int f'(x) e^x dx$$

$$= f(x) e^x - \int f'(x) e^x dx + \int f'(x) e^x dx$$

$$= e^x f(x) + C$$

- a) Find $\int e^x (\sin x + \cos x) dx$. (1)
- b) Find $\int e^x \left(\frac{x-1}{x^2} \right) dx$. (1)
- c) Find $\int e^x (1+x) dx$. (2)

OR

Find $\int_0^\pi e^x (\tan x + \sec^2 x) dx$. (2)

$$\int_a^b f(x) dx$$

Integration Paper Details – Analysis		
Objective Questions		
MCQ	1 marks x 8 question = 8	
Assertion Reason	1 marks x 2 question = 2	
Subjective		
Short type I	2 marks x 5 question = 10	
Short type II	3 marks x 4 question = 12	
Long Type	5 marks x 4 question = 20	
Case Studies Based		
Case 1	4 marks	
Case 2	4 marks	

Total = 60 marks Your Score =