Maxprap. Chapter wise Series Class 12th – Relation, Function and Inverse Trigonometry

Class : 12th **Subject: Mathematics**

Max. Marks : …... **Duration : …………**

NCERT Based Questions

Relation Example		
1.	Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1$ is congruent to $T_2\}$. Show that R is an equivalence relation.	**[2]**
2.	Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is perpendicular to $L_2\}$. Show that R is symmetric but neither reflexive nor transitive.	**[3]**
3.	Show that the relation R in the set {1, 2 ,3} given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.	**[1]**
4.	Show that the relation R in the set Z of integers given by R = {(a, b) : 2 divides a – b}is an equivalence relation.	**[3]**
5.	Let R be the relation defined on the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b): both a and b are either odd or even}. Show that R is an equivalence relation. Further, show that all the elements of the subset {1, 3, 5, 7 } are related to each other and all the elements of the subset {2, 4, 6} are related to each other, but no element of the subset {1, 3, 5, 7} is related to any element of the subset {2, 4, 6}.	**[2]**
6.	Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1$ is congruent to $T_2\}$. Show that R is an equivalence relation.	**[2]**
7.	Show that the relation R in the set {1, 2 ,3} given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.	**[1]**
8.	Show that the relation R in the set Z of integers given by R = {(a, b) : 2 divides a – b}is an equivalence relation.	**[3]**
9.	If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.	**[3]**
10.	Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v) if and only if xv = yu. Show that R is an equivalence relation.	**[2]**

Relation Exercise		
11.	Determine whether the below relations is reflexive, symmetric and transitive: Relation R in the set A = {1, 2, 3, ..., 13, 14} defined as R = {(x, y) : $3x - y = 0$}.	**[2]**
12.	Determine whether the below relation is reflexive, symmetric and transitive: Relation R in the set N of natural numbers is defined as R = {(x, y) : $y = x + 5$ and $x < 4$}	**[2]**
13.	Determine whether the below relation is reflexive, symmetric and transitive: Relation R in the set A = {1, 2, 3, 4, 5, 6} as R = {(x, y) : y is divisible by x}	**[2]**
14.	Determine whether the below relation is reflexive, symmetric and transitive: Relation R in the set Z of all integers defined as R = {(x, y) : $x - y$ is an integer}	**[2]**
15.	Show that the relation R in the set **R** of real numbers, defined as R = {(a, b): $a \leq b^2$ } is neither reflexive nor symmetric nor transitive.	**[2]**
16.	Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b): $b = a + 1$} is reflexive, symmetric or transitive.	**[3]**
17.	Show that the relation R in **R** defined as R = {(a, b) : $a \leq b$}, is reflexive and transitive but not symmetric.	**[2]**
18.	Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.	**[2]**
19.	Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : $\|a - b\|$ is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.	**[5]**
20.	Show the relation R in the set A = {$x \in Z : 0 \leq x \leq 12$}, given by R = {(a, b) : $\|a - b\|$ is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1in each case.	**[2]**
21.	Show the relation R in the set A = {$x \in Z : 0 \leq x \leq 12$}, given by R = {(a, b) : $a = b$}, is an equivalence relation. Find the set of all elements related to 1 in each case.	**[5]**
22.	Show that the relation R in the set A of points in a plane given by R = {(P, Q) : distance of the point P from the origin is same as the distance of the point Q	**[3]**

	from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P$\neq$ (0, 0) is the circle passing through P with origin as centre.	
23.	Show that the relation R defined in the set A of all polygons as R = {(P_1 , P_2) : P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4, and 5?	**[3]**
24.	Let L be the set of all lines in xy plane and R be the relation in L define as R = {(L_1 , L_2) : $L_1 \parallel L_2$ }. Show thatR is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.	**[5]**
25.	Let R be the relation in the set N given by R = {(a, b) : a = b – 2, b > 6}. a) (6, 8)$\in$ R b) (8, 7)$\in$ R c) (2, 4)$\in$ R d) (3, 8)$\in$ R	**[1]**
26.	Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is a) 1 b) 2 c) 3 d) 4	**[1]**
27.	Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is a) 4 b) 1 c) 3 d) 2	**[1]**
	Function Example	
28.	Show that the function f: N$\rightarrow$ N, given by f (x) = 2x, is one – one but not onto.	**[2]**
29.	Show that the function f: R$\rightarrow$ R, defined as f(x) = x^2 , is neither one – one nor onto.	**[2]**
30.	Show that the function f: N$\rightarrow$ N given by f (1) = f(2) = 1 and f (x) = x – 1,for every x > 2, is onto but not one – one.	**[2]**
31.	Prove that the function f: R$\rightarrow$ R, given by f(x) = 2x, is one – one and onto.	**[1]**
32.	Show that f : N$\rightarrow$ N, given by $f(x) = \begin{cases} x+1, & if x is odd \\ x-1, & if x is even \end{cases}$ is both one – one and onto.	**[3]**
33.	Show that an onto function f : {1, 2, 3}$\rightarrow$ {1, 2, 3} is always one – one.	**[1]**
	Function Exercise	
34.	Show that the function f : $R_+ \rightarrow R_+$ defined by $f(x) = \frac{1}{x}$ is one – one and onto, where R_+ is the set of all non – zero real numbers. Is the result true, if the domain R_+ is replaced by N with co – domain being same as R_+ ?	**[3]**

35.	Check the injectivity and surjectivity of the below function: f : N → N given by f(x) = x^{2}	[2]
36.	Check the injectivity and surjectivity of the below function: f : Z → Z given by f(x) = x^{2}	[2]
37.	Check the injectivity and surjectivity of the below function: f : N → N given by f(x) = x^{3}	[1]
38.	Check the injectivity and surjectivity of the below function: f : Z → Z given by f(x) = x^{3}	[1]
39.	Show that the Modulus Function f : R→ R, given by f(x) = |x|, is neither one – one nor onto, where |x| is x, if x is positive or 0 and |x| is – x, if x is negative.	[2]
40.	Show that the Signum Function f : R→ R, given by $f(x) = \begin{cases} 1, if\ x > 0 \\ 0, if\ x = 0 \\ 1, if\ x < 0 \end{cases}$ is neither one one nor onto.	[3]
41.	State whether the function is one – one, onto or bijective. Justify your answer. f: R→ R defined by f(x) = 3 – 4x.	[3]
42.	State whether the function is one – one, onto or bijective. Justify your answer. f: R→ R defined by f(x) = 1+ x^{2}	[2]
43.	If f: N→ N is defined by f(n) = $\begin{cases} \frac{n+1}{2}, ifnisodd \\ \frac{n}{2}, ifniseven \end{cases}$ for all n ∈ N. State whether the function f is bijective. Justify your answer.	[3]
44.	Let A = R – {3} and B = R – {1}. Consider the function f : A→ B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$ Is f one – one and onto? Justify your answer.	[5]
45.	Let $f: R \to R$ be defined as f (x) = 3x. Choose the correct answer. a) many – one onto b) one – one onto c) neither one – one nor onto d) one – one but not onto	[1]
46.	Show that the function f : R→ {x ∈ R : – 1 < x < 1} defined by $f(x) = \frac{x}{1+\|x\|}$, x ∈ R is one – one and onto function.	[5]
47.	Show that the function f : R→ R given by f(x) = x^{3} is injective.	[1]

Inverse Trigo Exercise		
48.	Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.	**[1]**
49.	Find the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$	**[1]**
50.	Show that ; $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x, -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$	**[2]**
51.	Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$	**[1]**

Inverse Trigo Example		
52.	Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.	**[1]**
53.	Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$	**[1]**
54.	Find the principal value of cosec^{-1} (2)	**[1]**
55.	Find the principal value of $\tan^{-1}(-\sqrt{3})$.	**[1]**
56.	Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$.	**[1]**
57.	Find the principal value of $\tan^{-1}(-1)$.	**[1]**
58.	Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$.	**[1]**
59.	Find the principal value of $\cot^{-1}(\sqrt{3})$.	**[2]**
60.	Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$	**[2]**
61.	Find the principal value of $\text{cosec}^{-1}(-\sqrt{2})$.	**[2]**
62.	Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$	**[3]**
63.	Find the value of $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$.	**[2]**
64.	$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to a) π b) $\frac{\pi}{3}$ c) $\frac{2\pi}{3}$ d) $-\frac{\pi}{3}$	**[1]**
65.	Prove that: $3\sin^{-1}x = \sin^{-1}(3x-4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$	**[2]**

66.	Write the function in the simplest form: $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}},\ 0 < x < \pi$	**[2]**
67.	Write the function in the simplest form: $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), -\frac{\pi}{4} < x < \frac{3\pi}{4}$	**[2]**
68.	Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$.	**[1]**
69.	Find the value of the expression $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$.	**[3]**
70.	$\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) 1	**[1]**
71.	$\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})$ is equal to a) π b) $-\frac{\pi}{2}$ c) $2\sqrt{3}$ d) 0	**[1]**