Maxprap. Chapter wise Series Class 12th – Differentiation and Higher Order Derivatives

Class : 12th **Subject: Mathematics**

Max. Marks : 60 **Duration : 2 Hrs**

Sections: A (MCQs), B (Short Answer-I), C (Short Answer-II), D (Long Answer), E (Case-Based).

General Instructions:

(i). All questions are compulsory.

(ii). This question paper contains 25 questions divided into five Sections A, B, C, D and E. (iii). Section A comprises of 10 MCQs of 1 mark each. Section B comprises of 5 questions of 2 marks each. Section C comprises of 4 questions of 3 marks each. Section D comprises of 4 question of 5 marks each and Section E comprises of 2 Case Study Based Questions of 4 marks each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted.

Section A

Questions 1 to 10 carry 1 mark each

1. If f(x) = $\begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & for -1 \le x < 0 \\ 2x^2+3x-2, & for 0 \le x \le 1 \end{cases}$ is continuous at x = 0, then k is equal to

a) - 3 b) - 2 c) - 4 d) - 1

2. The function f(x) = x – [x] , where [.] denotes the greatest integer function is

a) continuous at integer points only b) continuous everywhere

c) continuous at non - integer points only d) differentiable everywhere

3. The value of k for which f(x) = $\begin{cases} 3x+5, & x \ge 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is:

a) $-\frac{11}{4}$ b) 11 c) $\frac{11}{4}$ d) $\frac{4}{11}$

4. The function f(x) = |x| is

a) continuous and differentiable nowhere.

b) continuous and differentiable everywhere.

c) continuous everywhere, but differentiable everywhere except at x = 0.

d) continuous everywhere, but differentiable nowhere.

5. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is

a) $\frac{1+x}{1+\log x}$ b) $\frac{\log x}{(1+\log x)^2}$ c) not defined d) $\frac{1-\log x}{1+\log x}$

6. The point of discontinuity of the function f(x) = $\begin{cases} 2x+3, & if x \le 2 \\ 2x-3, & if x > 2 \end{cases}$ is

a) x = 0 b) x = 2 c) x = 1 d) x = - 1

7. If x = A cos 4t + B sin 4t, then $\frac{d^2x}{dt^2}$ is equal to

a) 16x b) x c) - 16x d) - x

8. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots}}} \ldots \infty$ then $\frac{dy}{dx}$ = ?

a) $\frac{\sin x}{(2y+1)}$ b) $\frac{\sin x}{(2y-1)}$ c) $\frac{\cos x}{(2y-1)}$ d) $\frac{\cos x}{(y-1)}$

Assertion(A)/Reason(R) Based (9-10): In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

9. Assertion (A): f(x) is continuous at x = a, iff $\lim_{x\to a}$ f(x) exists and equals to f(a).

Reason (R): If f(x) is continuous at a point, then $\frac{1}{f(x)}$ is also continuous at the point.

10. Assertion (A): The function f(x) = $\sqrt[3]{x}$ is continuous at all x except at x = 0.

Reason (R): The function f(x) = [x] is continuous at x = 2.99, where [ ] is the greatest integer function.

SECTION – B

Questions 11 to 15 carry 2 marks each

11. For what value of k is function $f(x) = \begin{cases} k(x^2 - 2x), & if\, x \le 0 \\ 4x + 1, & if\, x > 0 \end{cases}$ continuous at x = - 1?

12. Find all points of discontinuity of f,where f is defined by: $f(x) = \begin{cases} 2x + 3, & x \le 2 \\ 2x - 3, & x > 2 \end{cases}$

13. If $y = \left(\sqrt{x}\right)^{\left(\sqrt{x}\right)^{\left(\sqrt{x}\right)^{\ldots\infty}}}$, show that $\frac{dy}{dx} = \frac{y^2}{x(2-y\log x)}$.

14. Find $\frac{dy}{dx}$ of the function xy = e $^{(x-y)}$

15. If $\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)$ = a, show that $\frac{dy}{dx} = \frac{x(1-\tan a)}{y(1+\tan a)}$.

SECTION – C

Questions 16 to 19 carry 3 marks each

16. Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & if\ x < 0 \\ = a, & if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & if\ x > 0 \end{cases}$ .For what value of a, f is continuous at x = 0?

17. If x = $\tan\left(\frac{1}{a}\log y\right)$,then show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$.

18. If $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \log\sqrt{1-x^2}$, then prove that $\frac{dy}{dx} = \frac{\sin^{-1}x}{(1-x^2)^{3/2}}$.

19. If x = a cosθ + b sin θ , y = a sin θ - b cos θ ,then show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

SECTION – D

Questions 20 to 23 carry 5 marks each

20. If $x\sqrt{1-y} + y\sqrt{1+x} = 0$ prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

21. Find $\frac{dy}{dx}$, if $y = (x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

22. If $e^y(x+1) = 1$ show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

23. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

SECTION – E (Case Study Based Questions)

Questions 24 & 25 carry 4 marks each

24. Read the following text carefully and answer the questions that follow:

The function f(x) will be discontinuous at x = a if f(x) has

- Discontinuity of first kind : $\lim_{h\to 0} f(a-h)$ and $\lim_{h\to 0} f(a+h)$ both exist but are not equal. If is also known as irremovable discontinuity.
- Discontinuity of second kind : If none of the limits $\lim_{h\to 0} f(a-h)$ and $\lim_{h\to 0} f(a+h)$ exist.
- Removable discontinuity: $\lim_{h\to 0} f(a-h)$ and $\lim_{h\to 0} f(a+h)$ both exist and equal but not equal to f(a).

a) What can you tell me about the behaviour of the function $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & for x \neq 3 \\ 4, & for x = 3 \end{cases}$, at x = 3irremovable orremovablediscontinuity? (1)

b) What can you tell me about the behaviour of the function f(x) = $\begin{cases} x+2, & if x \leq 4 \\ x+4, & if x > 4 \end{cases}$ at x = 4irremovable discontinuity? (1)

c) What can you tell me about the behaviour of the function f(x)=$\begin{cases}\frac{x^2-4}{x-2}, & for x \neq 2 \\ 5, & for x = 2\end{cases}$,at x = 2irremovable discontinuity? (2)

OR

What can you tell me about the behaviour of the function f(x) = $\begin{cases}\frac{x-|x|}{x}, & x \neq 0 \\ 2, & x = 0\end{cases}$, at x = 0 irremovable discontinuity? (2)

25. Read the following text carefully and answer the questions that follow:

Derivative of y = f(x) w.r.t. x (if exists) is denoted by $\frac{dy}{dx}$ or f'(x) and is called the first - order derivative of y. If we take the derivative of $\frac{dy}{dx}$ again, then we get $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ or f''(x) and is called the second - order derivative of y. Similarly, $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$ is denoted and defined as $\frac{d^3y}{dx^3}$ or f'''(x) and is known as the third - order derivative of y and so on.

a) If y = $\tan^{-1}\left(\frac{\log(e/x^2)}{\log(ex^2)}\right)$ + $\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, then find the value of $\frac{d^2y}{dx^2}$. (1)

b) If u = x^2 + y^2 and x = s + 3t, y = 2s - t, then find the value of $\frac{d^2u}{ds^2}$. (1)

c) If f(x) = 2 log sin x, then find f'(x). (2)

OR

If f(x) = e^x sinx, then find f''(x). (2)

Differentiation and Higher Order Derivatives Paper Details – Analysis		
Objective Questions		
MCQ	1 marks x 8 question = 8	
Assertion Reason	1 marks x 2 question = 2	
Subjective		
Short type I	2 marks x 5 question = 10	
Short type II	3 marks x 4 question = 12	
Long Type	5 marks x 4 question = 20	
Case Studies Based		
Case 1	4 marks	
Case 2	4 marks	

Total = 60 marks Your Score =