Maxprap. Chapter wise Series Class 12th – Application of Derivatives

Class : 12th **Subject: Mathematics**
Max. Marks : 60 **Duration : 2 Hrs**

Sections: A (MCQs), B (Short Answer-I), C (Short Answer-II), D (Long Answer), E (Case-Based).

General Instructions:
(i). All questions are compulsory.
(ii). This question paper contains 23 questions divided into five Sections A, B, C, D and E. (iii). Section A comprises of 10 MCQs of 1 mark each. Section B comprises of 5 questions of 2 marks each. Section C comprises of 4 questions of 3 marks each. Section D comprises of 4 question of 5 marks each and Section E comprises of 2 Case Study Based Questions of 4 marks each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted.

Section A

Questions 1 to 10 carry 1 mark each

1. The function f(x) = tan x – x
a) always decreases b) never increases
c) sometimes increases and sometimes decreases d) always increases

2. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in [0, 9] is
a) 126 b) 160 c) 135 d) 0

3. maximum value of f(x)=sinx in $[\pi, 2\pi]$ is
a) $\frac{1}{\sqrt{2}}$ b) 1 c) –1 d) 0

4. Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if
a) $f'(x) > 0, \forall x \in (a,b)$ b) $f'(x) < 0, \forall x \in (a,b)$
c) $f(x) > 0, \forall x \in (a,b)$ d) $f'(x) = 0, \forall x \in (a,b)$

5. The maximum value of $\left(\frac{\log x}{x}\right)$ is
a) 1 b) $\frac{2}{e}$ c) $\left(\frac{1}{e}\right)$ d) e

6. The rate of change of area of square is 40 cm^2 /s. What will be the rate of change of side if the side is 5 cm.
a) 6 cm/s b) 4 cm/s c) 8 cm/s d) 2 cm/s

7. The maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$ is:
a) 9 b) 0 c) 12 d) 15

8. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when x = 15 is

a) 90 b) 126 c) 96 d) 116

Assertion(A)/Reason(R) Based (9-10): In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

9. Assertion (A): $f(x) = \tan x - x$ always increases.

Reason (R): Any function y = f(x) is increasing if $\frac{dy}{dx} > 0$.

10. Assertion (A): The rate of change of area of a circle with respect to its radius r when r = 6 cm is 12π cm^2/cm.

Reason (R): Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

SECTION – B

Questions 11 to 15 carry 2 marks each

11. Find the interval in which the function $f(x) = x^3 - 6x^2 + 9x + 15$ is increasing or decreasing.

12. Water is running into an inverted cone at the rate of π cubic metres per minute. The height of the cone is 10 metres, and the radius of its base is 5 m. How fast the water level is rising when the water stands 7.5 m below the base.

13. A stone is dropped into a quiet lake and the waves move in circles. If the radius of a circular wave increases at the rate of 4 cm/sec, find the rate of increase in its area at the instant when its radius is 10 cm.

14. Find the intervals in which the function f(x) = $\log(1 + x) - \frac{2x}{2+x}$ is increasing or decreasing.

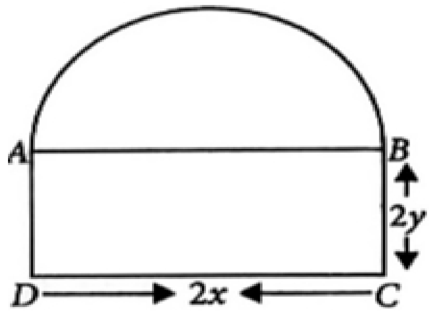
15. A men of height 2m walks at a uniform speed of 5km/h away from a lamp, past which is 6m high. Find the rate at which the lengths of his shadow increase.

SECTION – C

Questions 16 to 19 carry 3 marks each

16. Show that the height of the cylinder of maximum volume that can be, inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$ .Find the volume of the largest cylinder inscribed in a sphere of radius R.

17. A figure consists of a semi – circle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum.



18. A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum? Also, find this maximum volume.

19. A wire of length 25 m is to be cut into two pieces. One of the wires is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is minimum?

SECTION – D

Questions 20 to 23 carry 5 marks each

20. Prove that the semi – vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$.

21. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

22. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

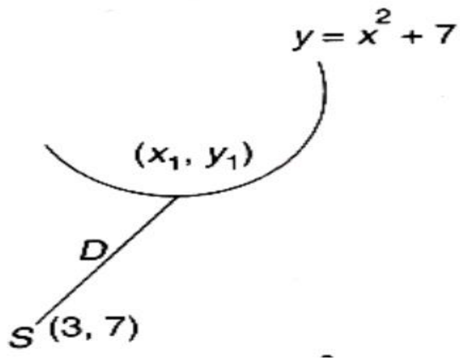
23. Show that for all the rectangles of given area, the square has the smallest perimeter.

SECTION – E (Case Study Based Questions)

Questions 24 & 25 carry 4 marks each

24. Read the following text carefully and answer the questions that follow:

An Apache helicopter of the enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7) want to shoot down the helicopter when it is nearest to him.



a) If P (x_1, y_1) be the position of a helicopter on curve $y = x^2 + 7$, then find distance D from P to soldier place at (3, 7). (1)

b) Find the critical point such that distance is minimum. (1)

c) Verify by second derivative test that distance is minimum at (1, 8). (2)

OR

Find the minimum distance between soldier and helicopter? (2)

25. Read the following text carefully and answer the questions that follow:

The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

a) Find the rate of growth of the plant with respect to sunlight. (1)

b) What is the number of days it will take for the plant to grow to the maximum height? (1)

c) Verify that height of the plant is maximum after four days by second derivative test and find the maximum height of plant. (2)

OR

What will be the height of the plant after 2 days? (2)

Application of Derivatives Paper Details – Analysis		
Objective Questions		
MCQ	1 marks x 8 question = 8	
Assertion Reason	1 marks x 2 question = 2	
Subjective		
Short type I	2 marks x 5 question = 10	
Short type II	3 marks x 4 question = 12	
Long Type	5 marks x 4 question = 20	
Case Studies Based		
Case 1	4 marks	
Case 2	4 marks	

Total = 60 marks   Your Score =